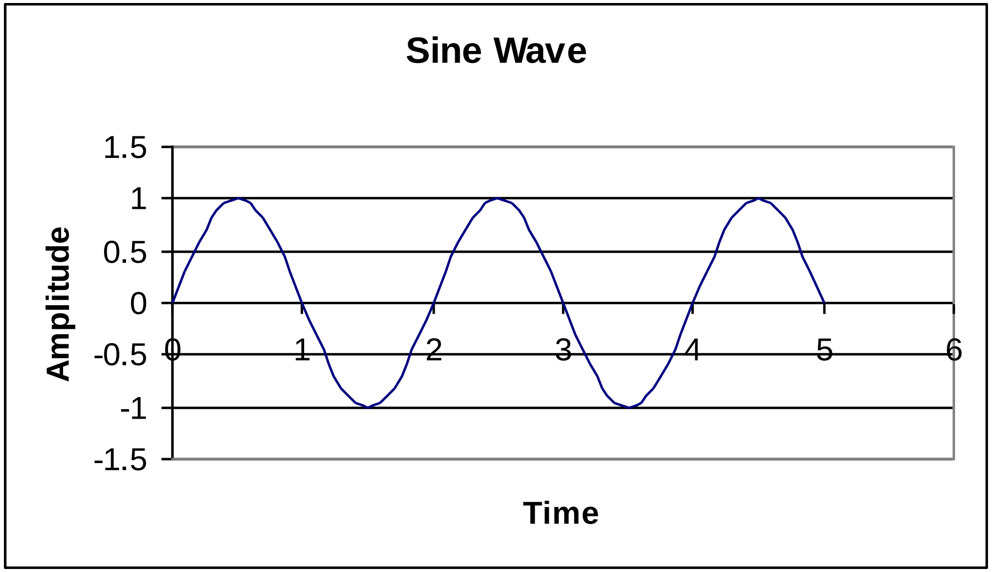
Sampling Theory

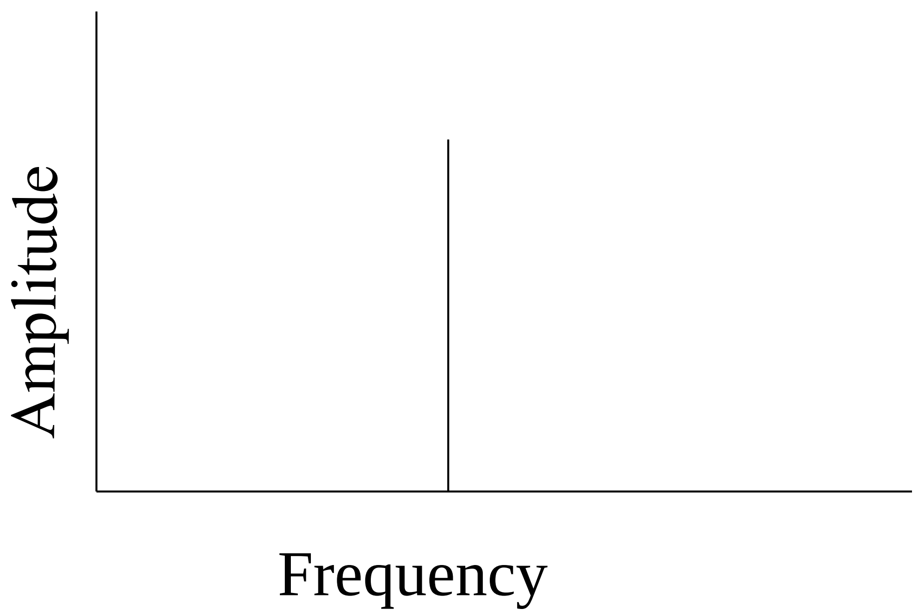
Time domain

- Present a recurring phenomena as amplitude vs. time
  - ➢Sine Wave

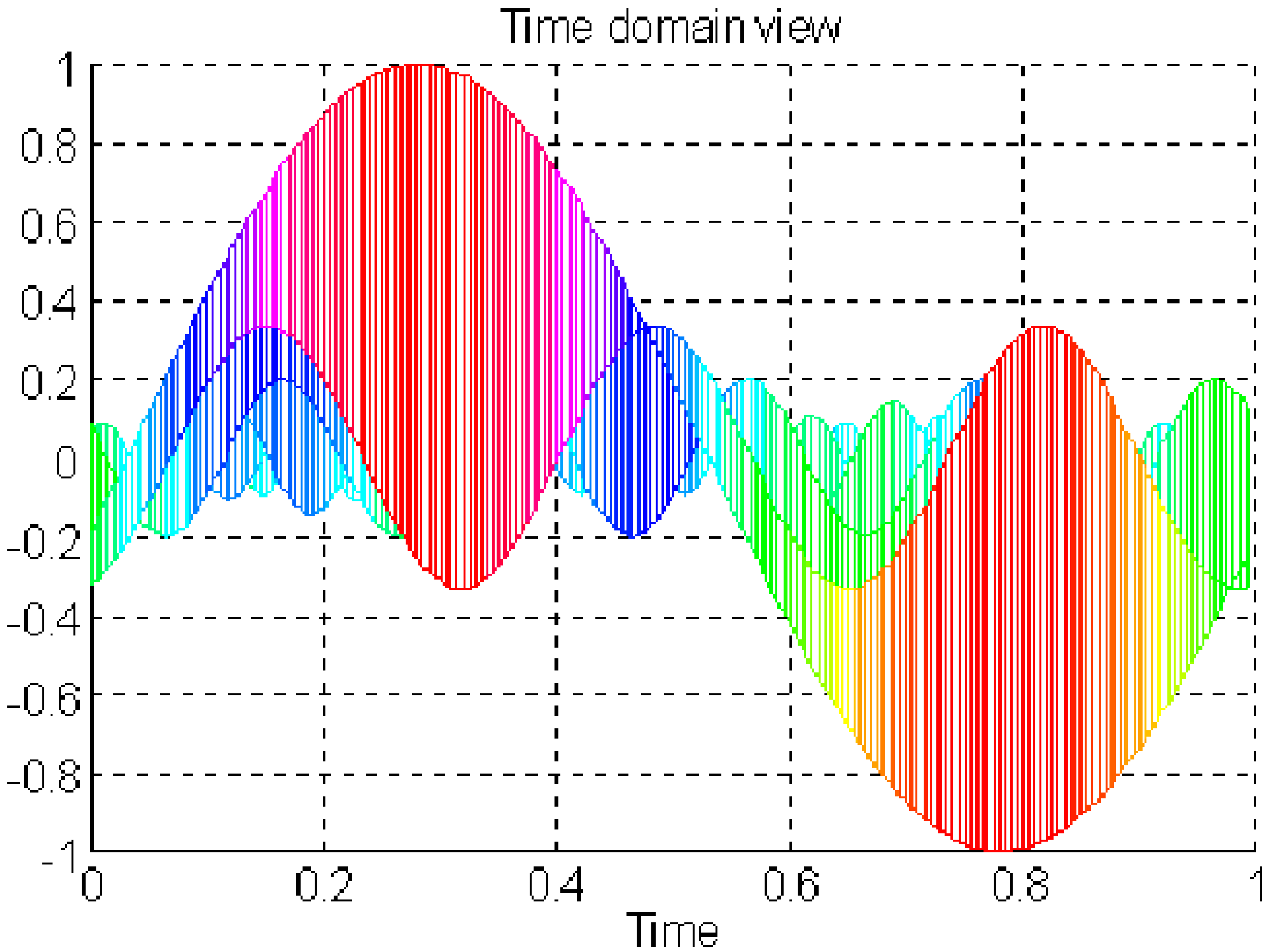


Frequency domain

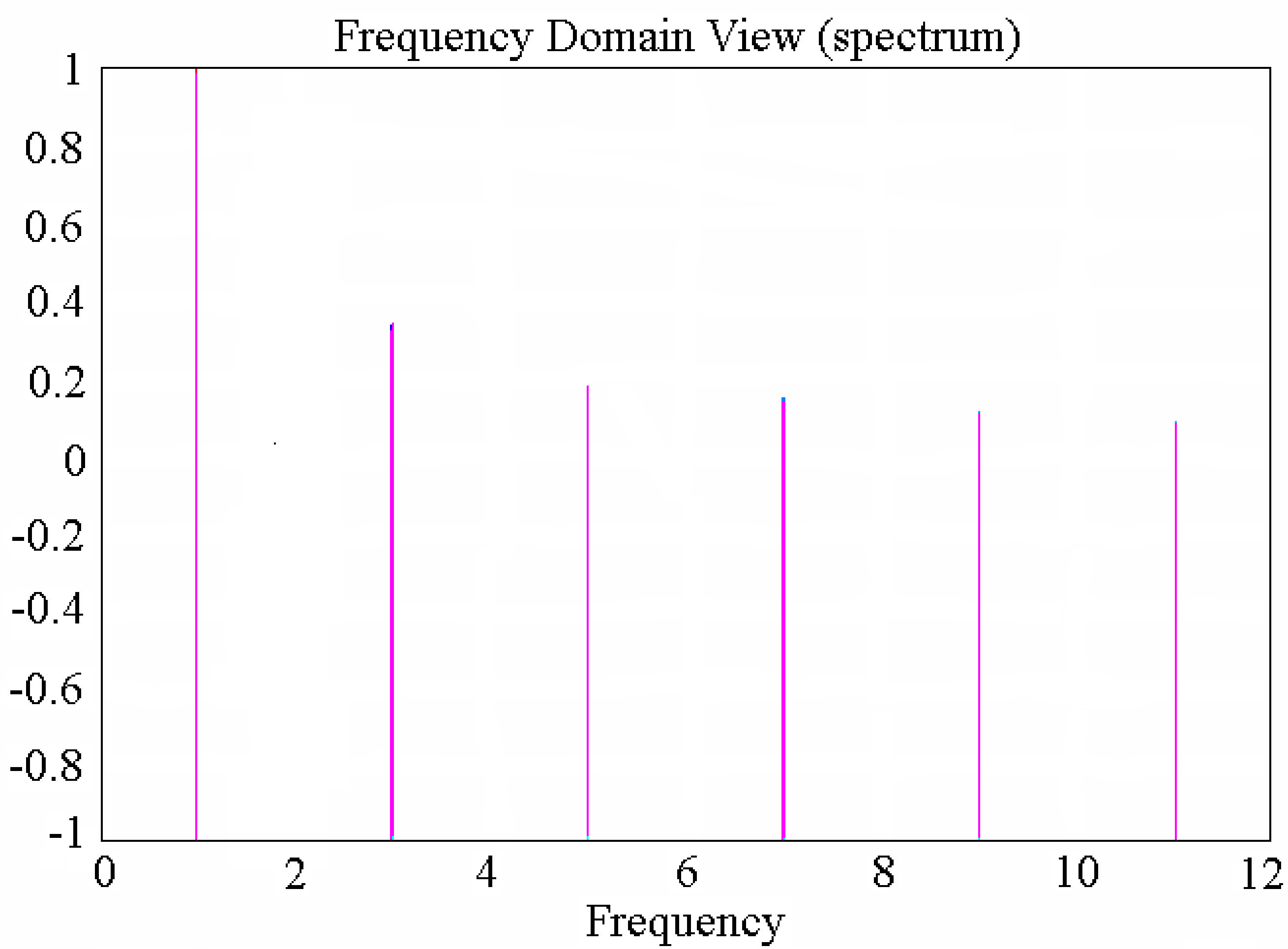
- Present recurring phenomena as amplitude vs. frequency
- Same sine wave looks like –



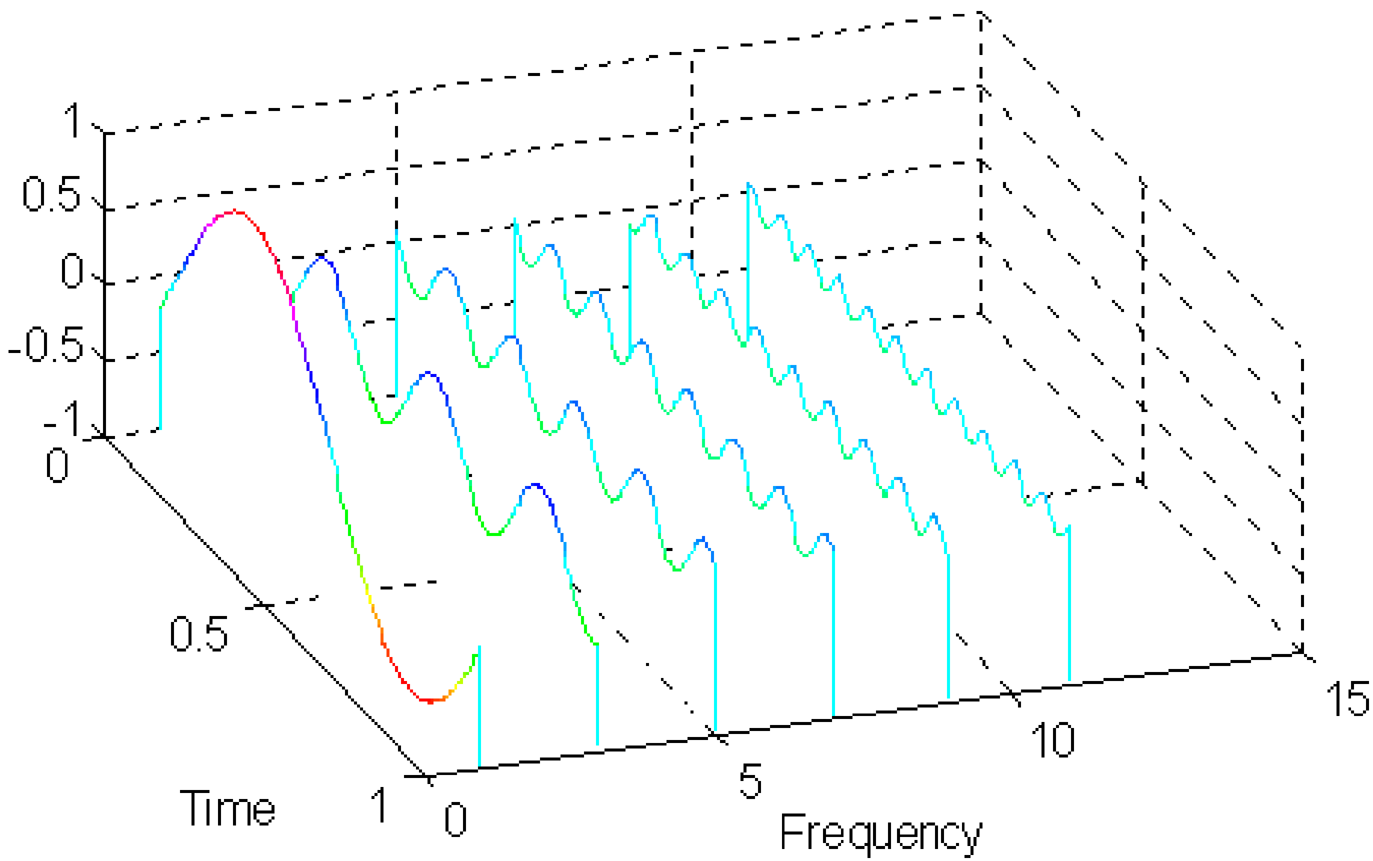
Multiple Waves



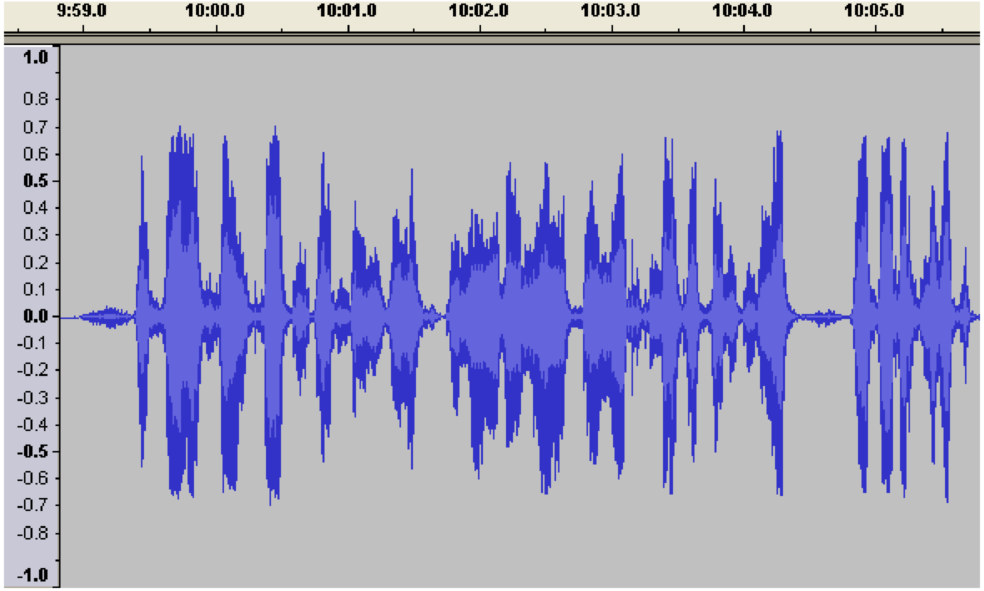
Multiple Waves



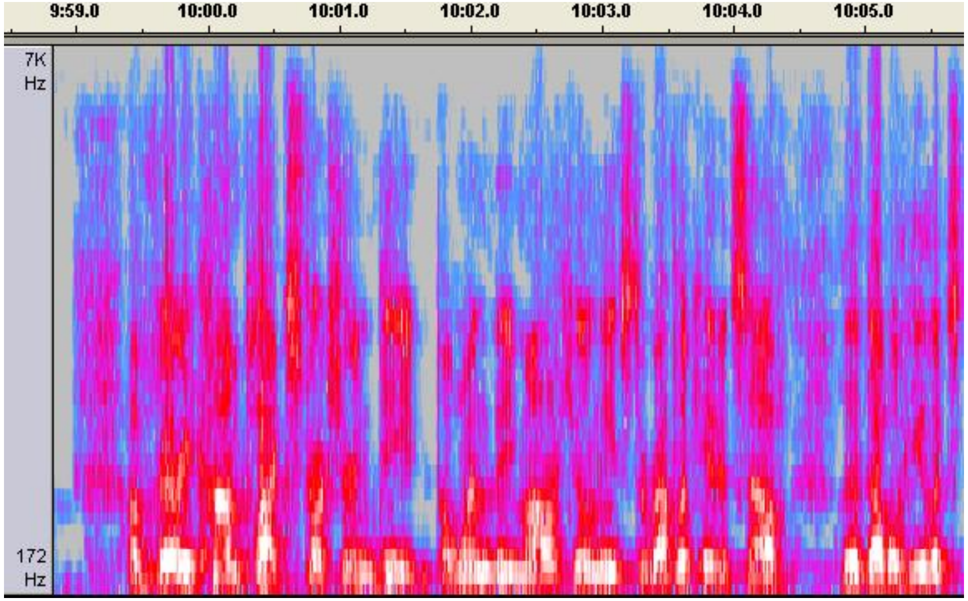
Both Domains



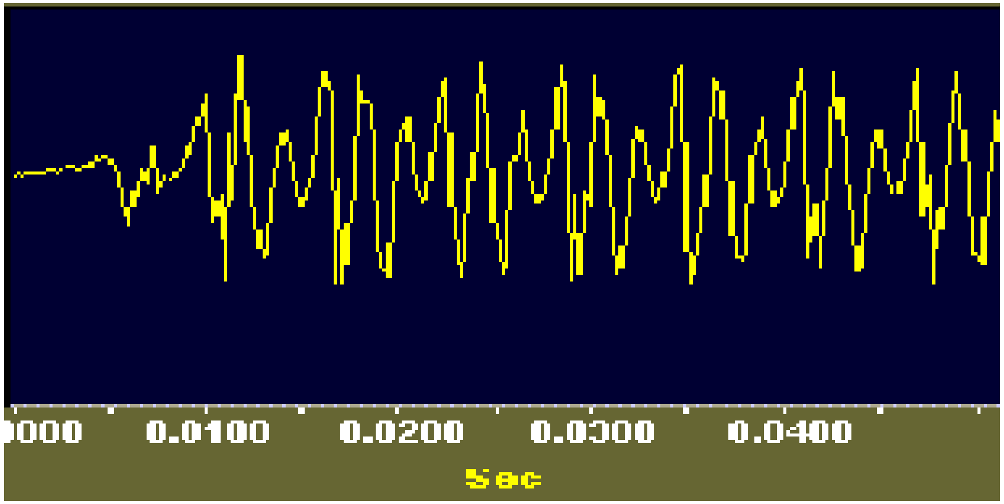
Voice in both Domains



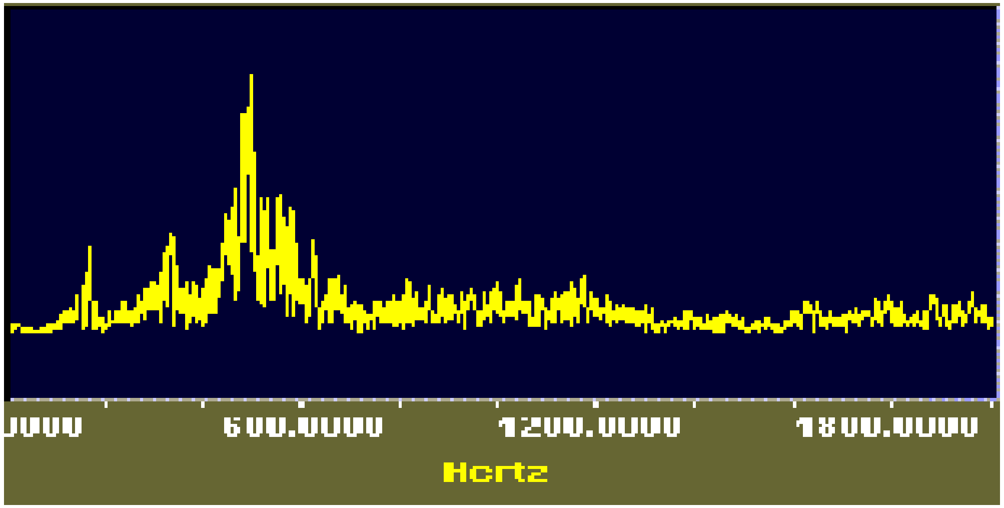
Voice in the Time Domain



Voice in the Frequency Domain



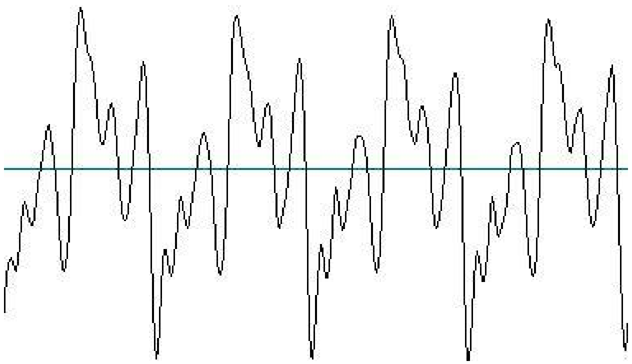
Voice signal
in time domain



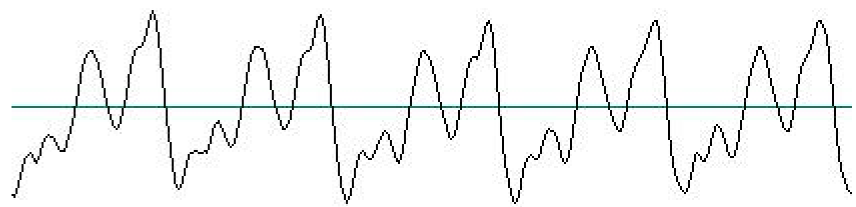
Voice signal
in freq. domain

Harmonics

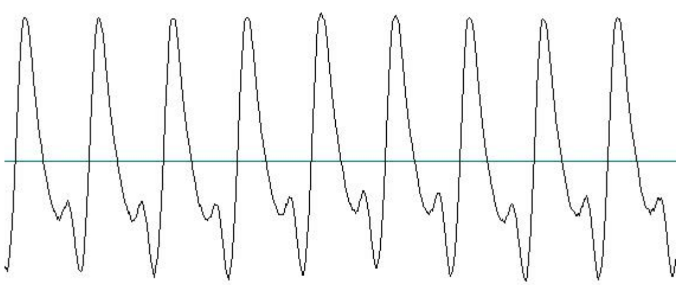
- See [Spreadsheet]



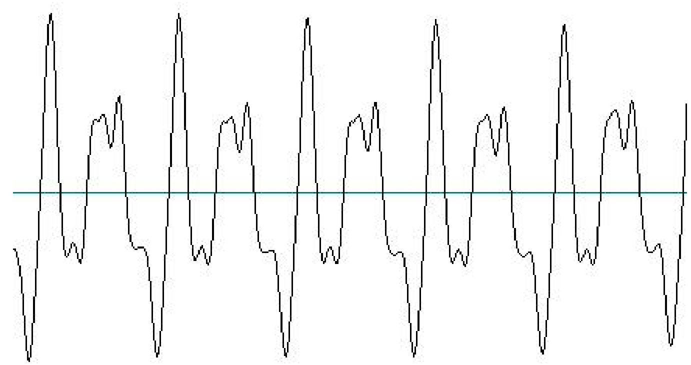
Clarinet



Flute



Horn



Guitar

Fourier Analysis

Jean Baptiste Joseph Fourier

- The eardrum responds to a sum of all the waves arriving at a particular instant. Yet the individual sounds are "heard."
- Any waveform is composed of an infinite number of simple sine waves of various frequencies and amplitudes.

Diatonic C Major Scale

Note	Letter Name	Frequency (Hz)	Frequency ratio	Interval
do	C	264		
			9/8	Whole
re	D	297		
			10/9	Whole
mi	E	330		
			16/15	Half
fa	F	352		
			9/8	Whole
sol	G	396		
			10/9	Whole
la	A	440		
			9/8	Whole
ti	B	495		
			16/15	Half
do	C	528		

The Keyboard

- Virtual Keyboard

EQUITEMPERED SCALE

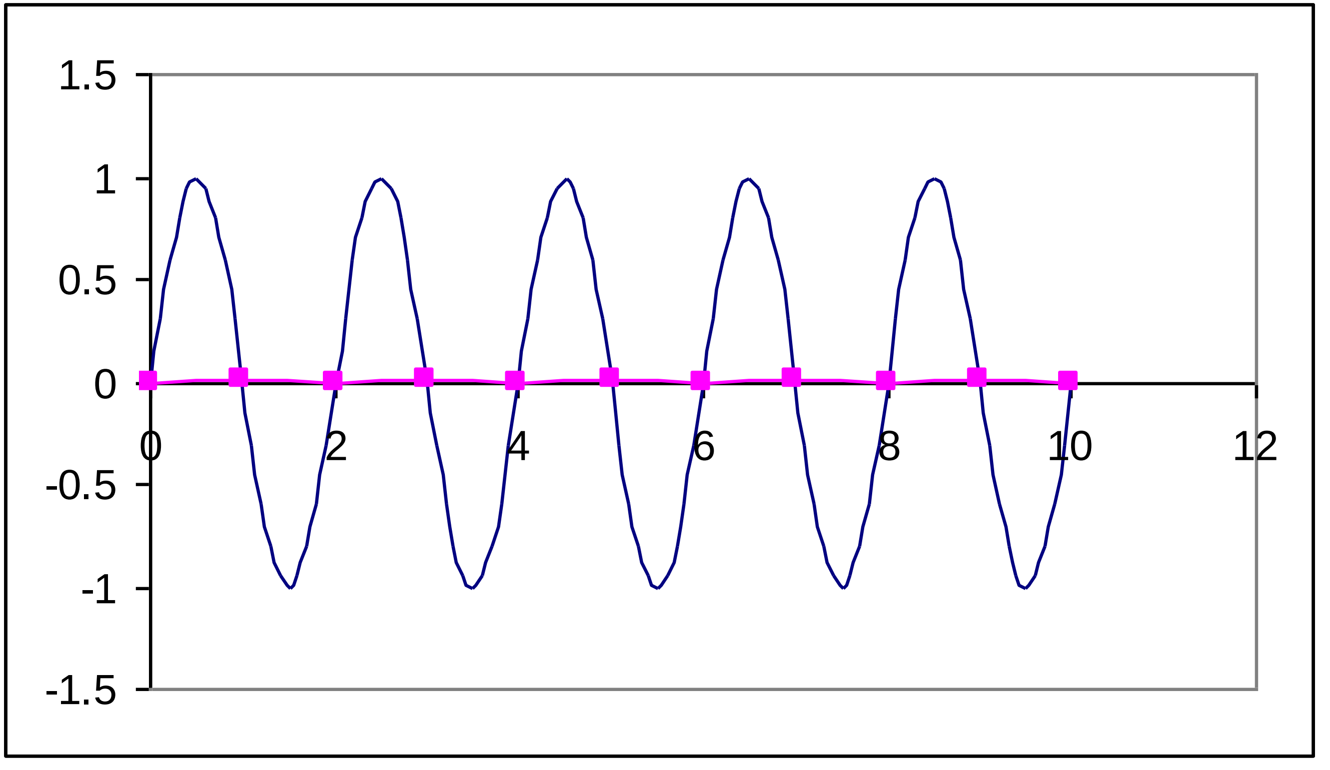
THE INTERVAL IS THE TWELTH ROOT OF TWO (1.059)

NOTE	FREQUENCY(HZ)	NOTE	FREQUENCY(HZ)
C	262	G	392
C#	277	G#	415
D	294	A	440
D#	311	A#	466
E	330	B	494
F	349	C'	524
F#	370		

Digitizing the Sound

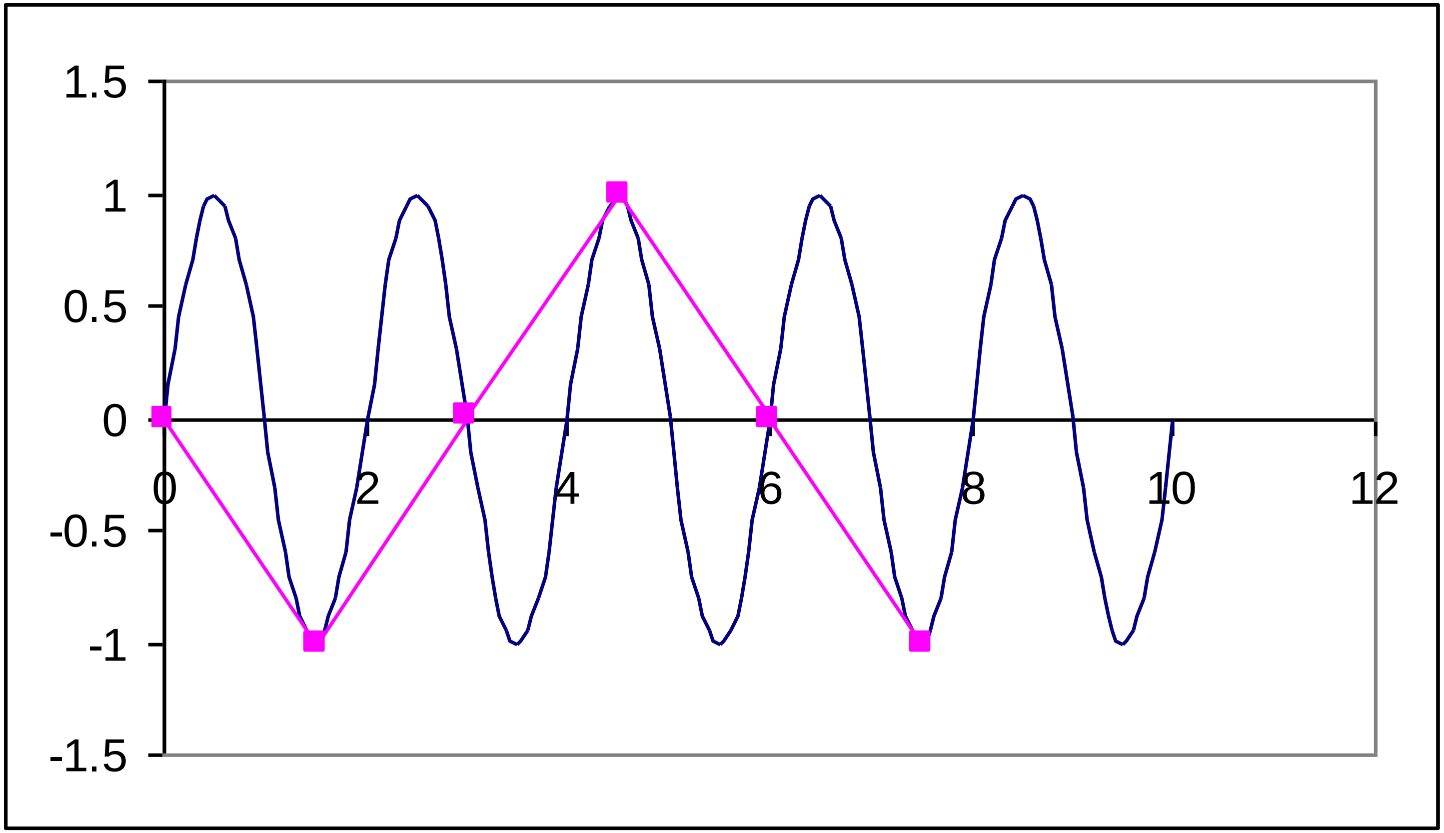
- We want to reconstruct the sound digitally
- How often must we obtain a sample to faithfully reproduce the sound?

Poor Sampling



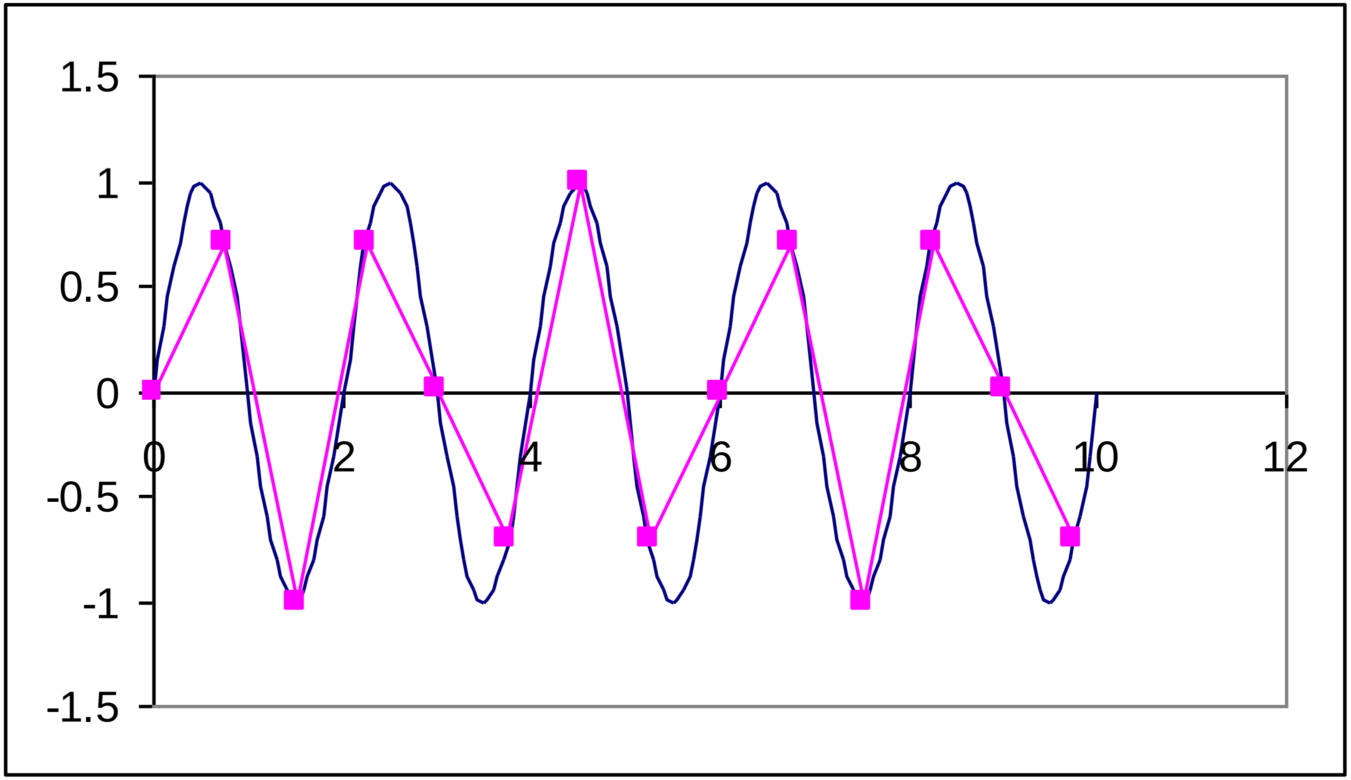
Sampling Frequency = 1/2 X Wave Frequency

Even Worse



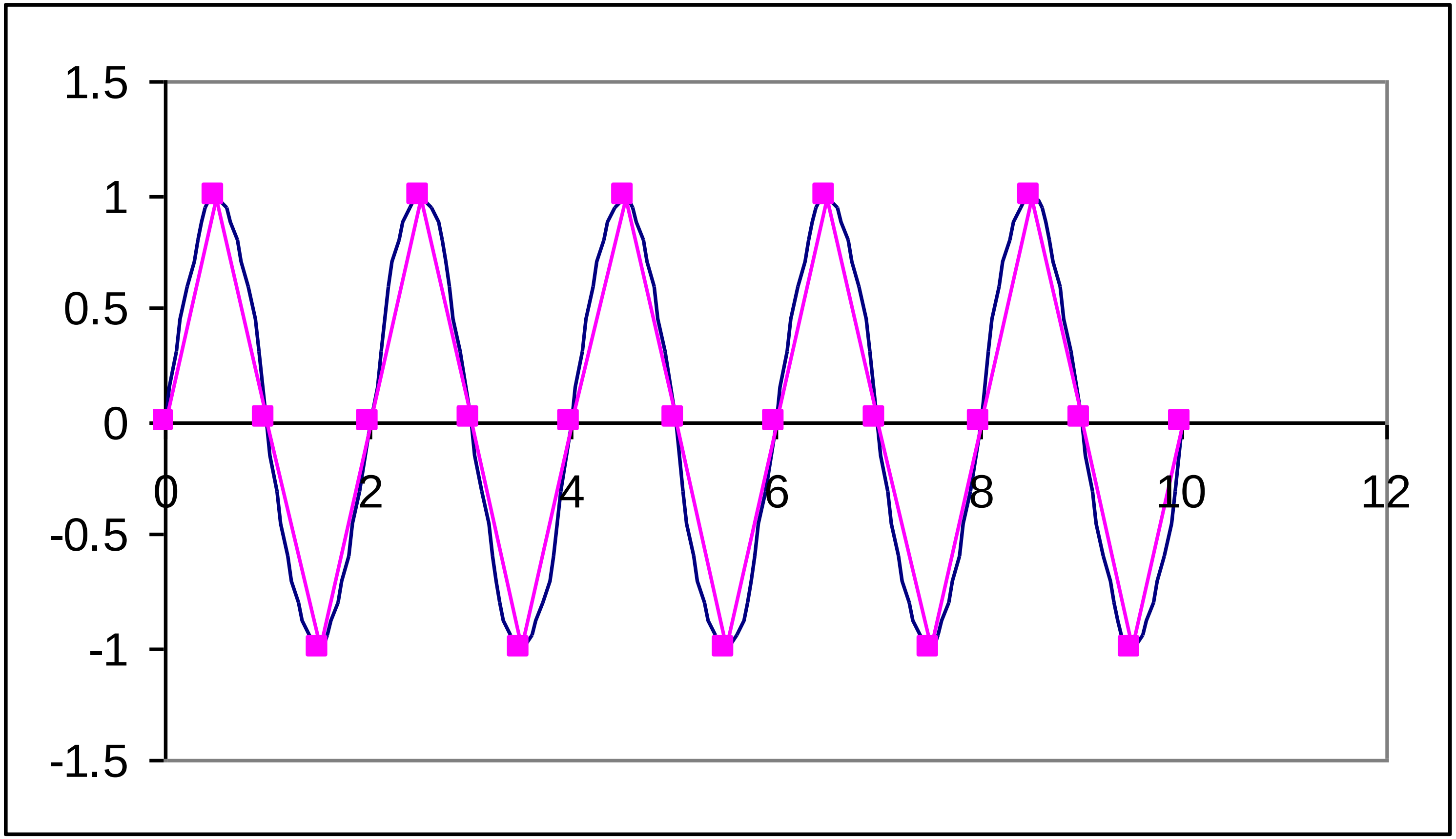
Sampling Frequency = 1/3 X Wave Frequency

Higher Sampling Frequency



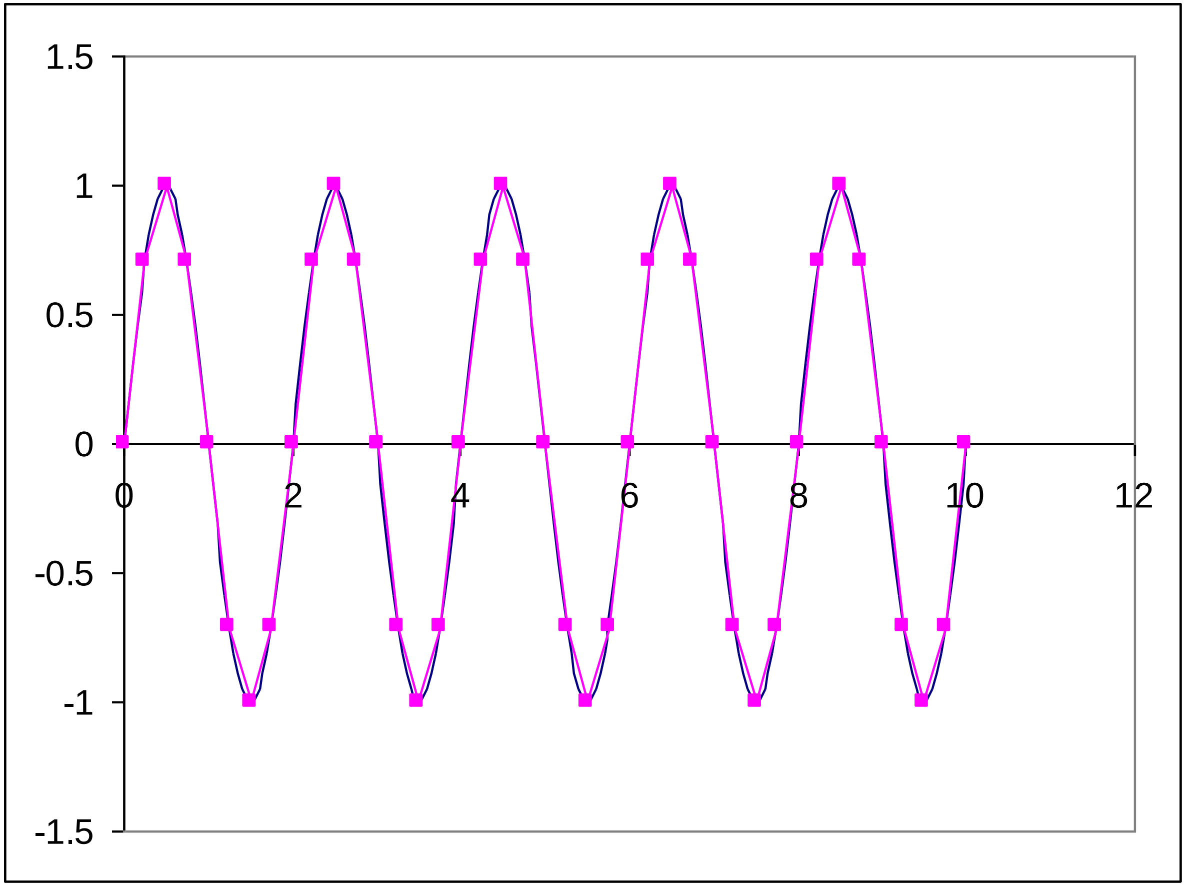
Sampling Frequency = 2/3 Wave Frequency

Getting Better



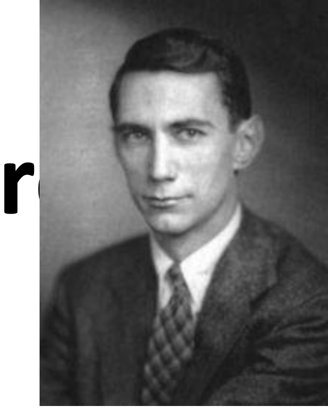
Sampling Frequency = Wave Frequency

Good Sampling



Sampling Frequency = 2 X Wave Frequency

quist-Shannon Sampling Theor



- A sampled time signal must not contain components at frequencies above half the sampling rate (The so-called Nyquist frequency)

- The highest frequency which can be accurately represented is one-half of the sampling rate

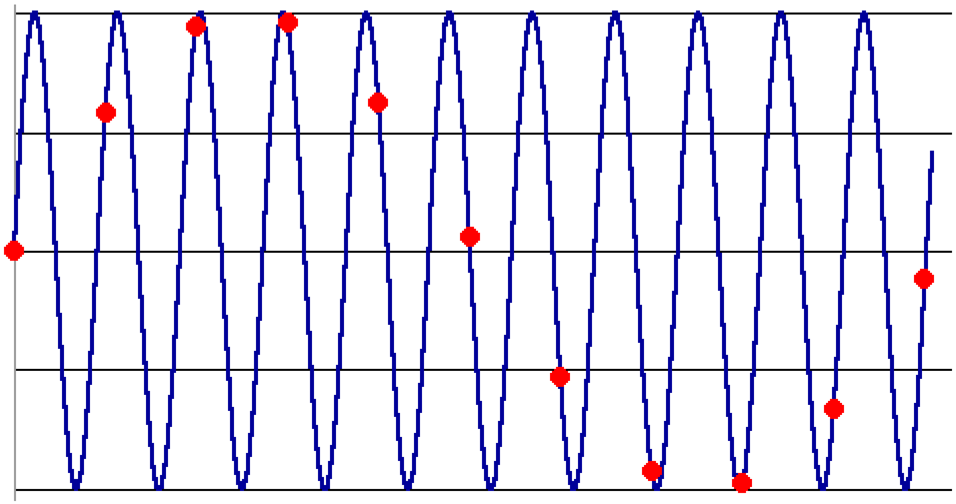
Range of Human Hearing

- 20 – 20,000 Hz
- We lose high frequency response with age
- Women generally have better response than men
- To reproduce 20 kHz requires a sampling rate of 40 kHz
  - Below the Nyquist frequency we introduce aliasing

Effect of Aliasing

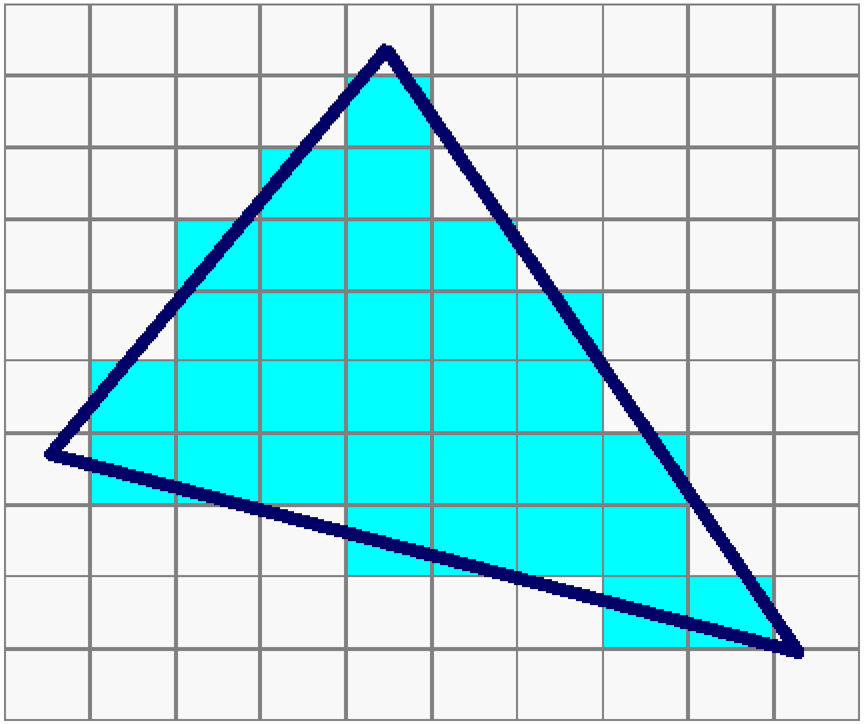
- Fourier Theorem states that any waveform can be reproduced by sine waves.
- Improperly sampled signals will have other sine wave components.

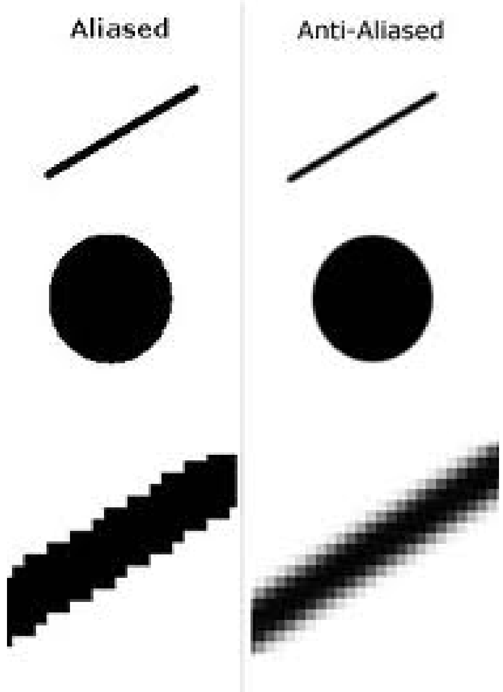
Example of Aliasing



- The blue is the original signal
- The red dots are the samples
  - Obviously, the red is a poor representation of the signal

Another Example
Spatial Aliasing



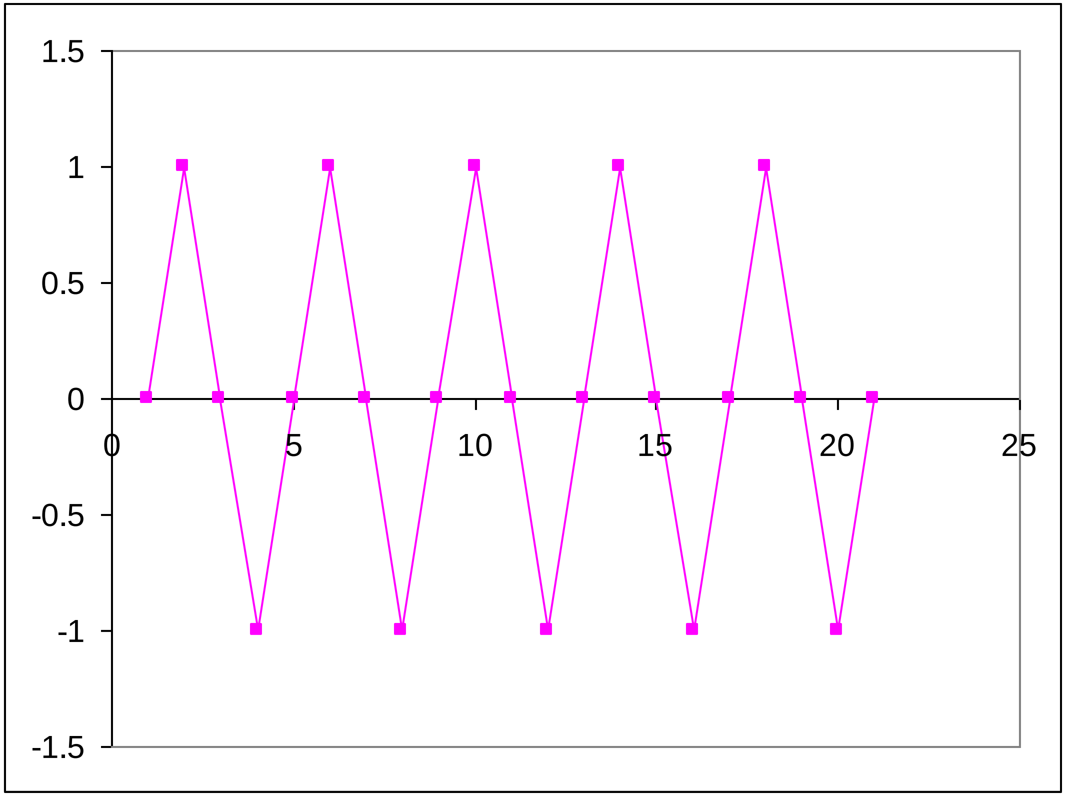


- Correcting for aliasing is called anti-aliasing

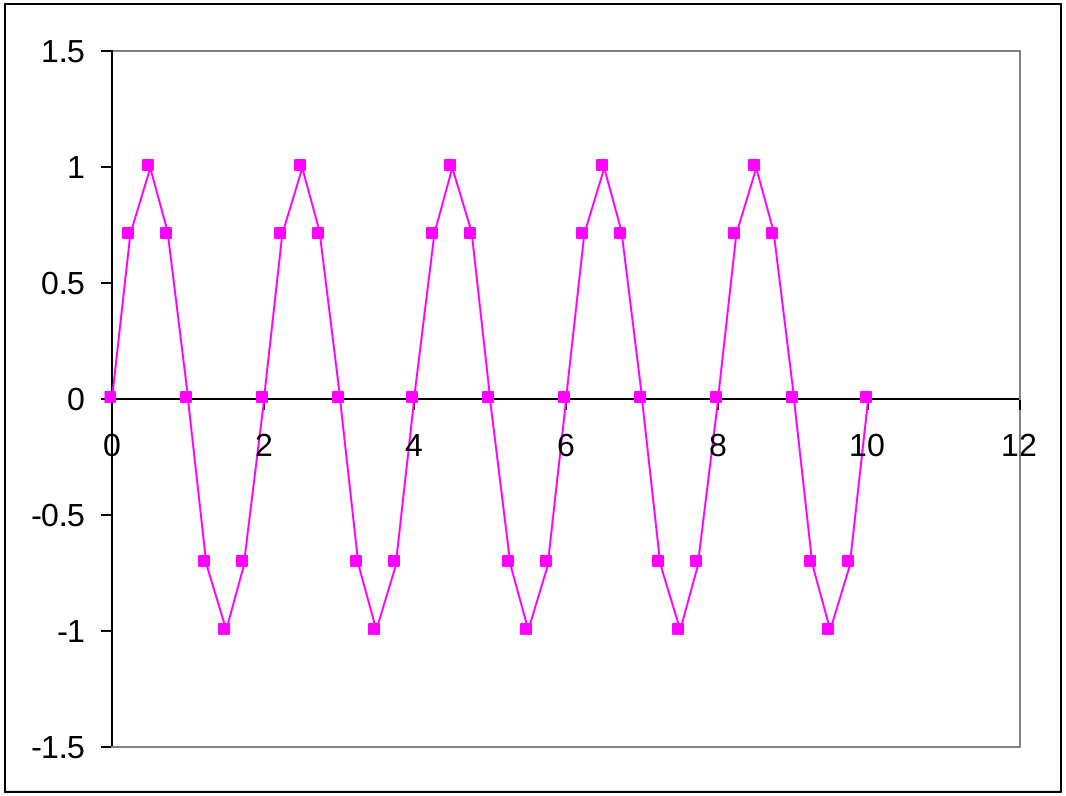
Temporal Aliasing

- Wagon Wheel
- Helicopter

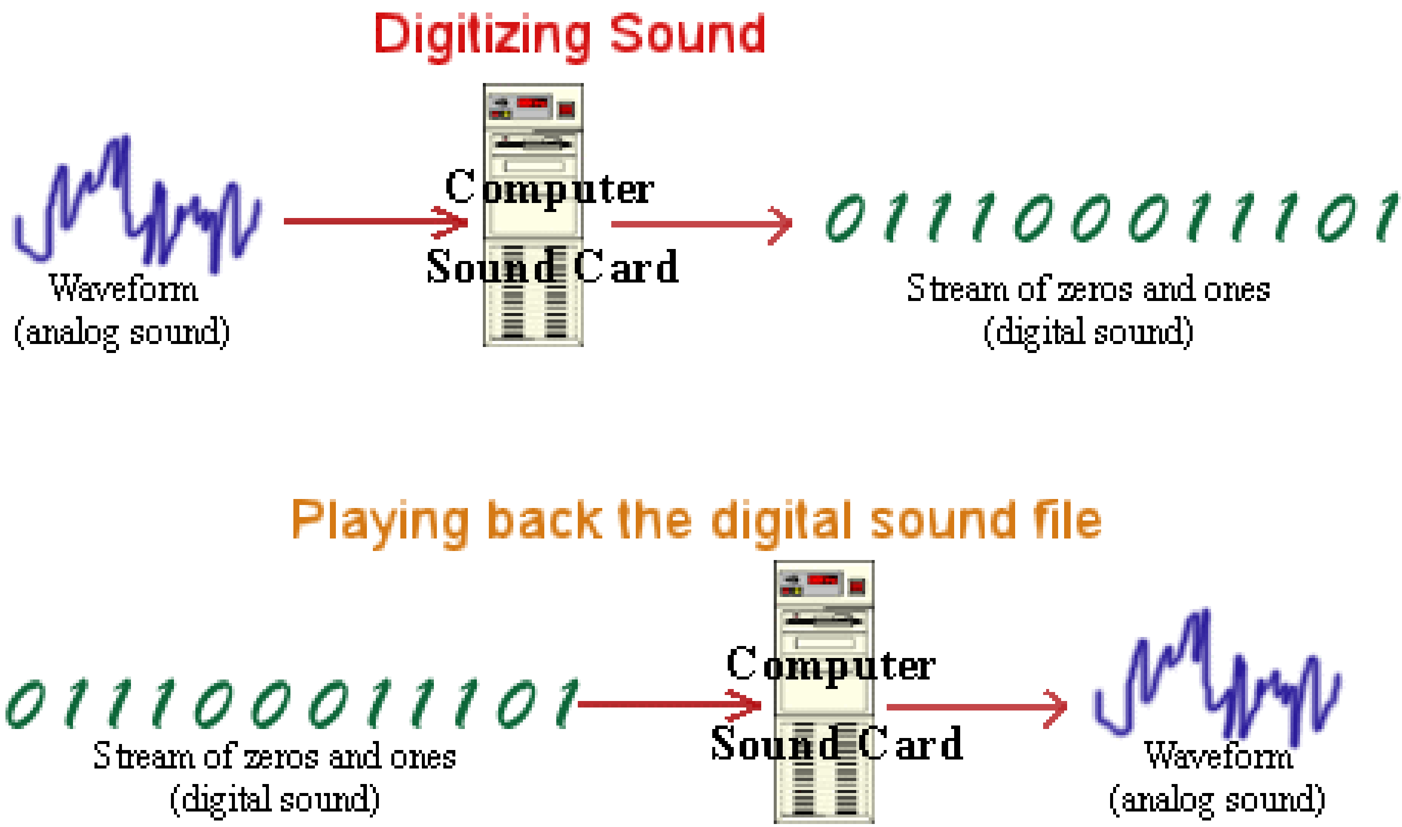
Half the Nyquist Frequency



Nyquist Frequency



Digitizing


Digitizing Sound
Computer
Sound Card
0 1 1 1 0 0 0 1 1 1 0 1
Waveform
(analog sound)
Stream of zeros and ones
(digital sound)
Playing back the digital sound file
0 1 1 1 0 0 0 1 1 1 0 1
Computer
Sound Card
Stream of zeros and ones
(digital sound)
Waveform
(analog sound)

Key Parameters

- Sampling frequency
  - 11.025kHZ or 22.05kHZ or 44.1kHZ
- Number of bits per sample
  - 8 bits (256 levels) or 16 bits (65,536 levels)
  - 44.1 kHz at 8 bits gives 172.3 Hz/bit (almost an octave) – [44,100 Hz/256 = 172.3 Hz/bit]
  - 44.1 kHz at 16 bits gives 0.67 Hz/bit – [44,100 Hz/65536 = 0.67 Hz/bit]

Digital Voice Telephone Transmission (DS0)

- Voice data for telephony purposes is limited to frequencies less than 4,000 Hz.
- According to Nyquist, it would take 8,000 samples/sec (2 times 4,000) to capture a 4,000 Hz signal perfectly.
- Generally, one byte is recorded per sample (256 levels). One byte is eight bits of binary data.
- (8 bits * 8,000 samples/sec = 64K bps) over a circuit.

T-1 Transmisson

- T carrier circuits are designed around this requirement, since they are primarily designed to carry analog voice signals that have been digitalized.
- For example, look at the DS-1 signal (digital signal 1) which passes over a T-1 circuit. For DS-1 transmissions, each frame contains 8 bits per channel and there are 24 channels. Also, one "framing bit" is required for each of the 24 channel frames.

T-1 Transmissons

- (24 channels * 8 bits per channel) + 1 framing bit = 193 bits per frame.
  193 bits per frame * 8,000 "Nyquist" samples = 1,544,000 bits per second.
- And it just so happens that the T-1 circuit is 1.544 Mbps.--not a coincidence. Each of the 24 channels in a T-1 circuit carries 64Kbps.

Standards

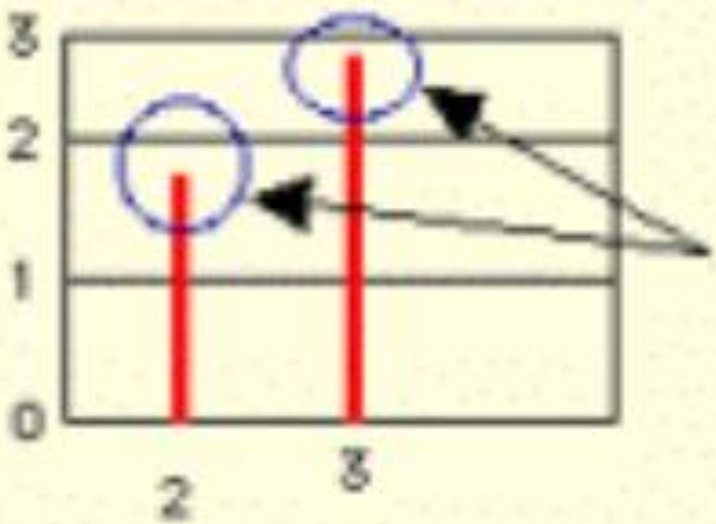
- DS0 – also called timeslots - 64 kilobits per second (telephone modem)
- ISDN - Two DS0 lines plus signaling (16 kilobytes per second), or 128 kilobits per second
- T1 - 1.544 megabits per second (24 DS0 lines)
- T3 - 43.232 megabits per second (28 T1s)
- OC3 - 155 megabits per second (84 T1s)
- OC12 - 622 megabits per second (4 OC3s)
- OC48 - 2.5 gigabits per seconds (4 OC12s)
- OC192 - 9.6 gigabits per second (4 OC48s) [Internet 2]

How Fast is It?

Downloading of the movie Matrix, which is about 136 minutes on DVD

- Standard telephone modem it took 171 hours
- ISDN it took almost 74 hours
- DSL or Cable Modem took 25 hours
- T1 line took about 6.5 hours
- Internet2 about 30 seconds (see Columbia Center)

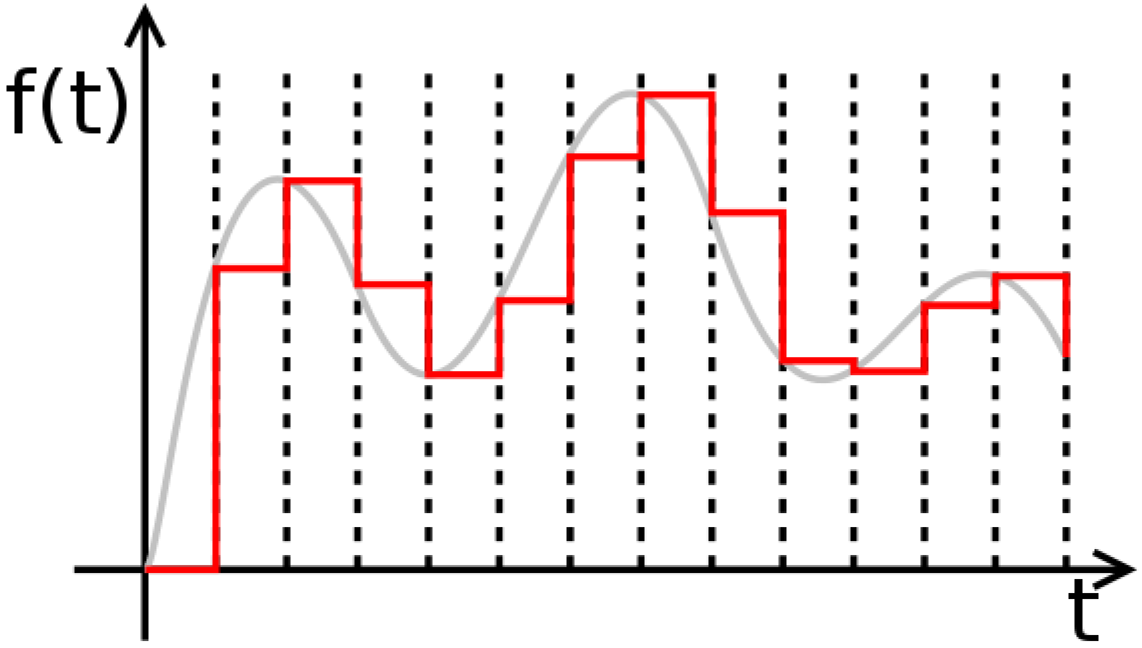
Quantization Error



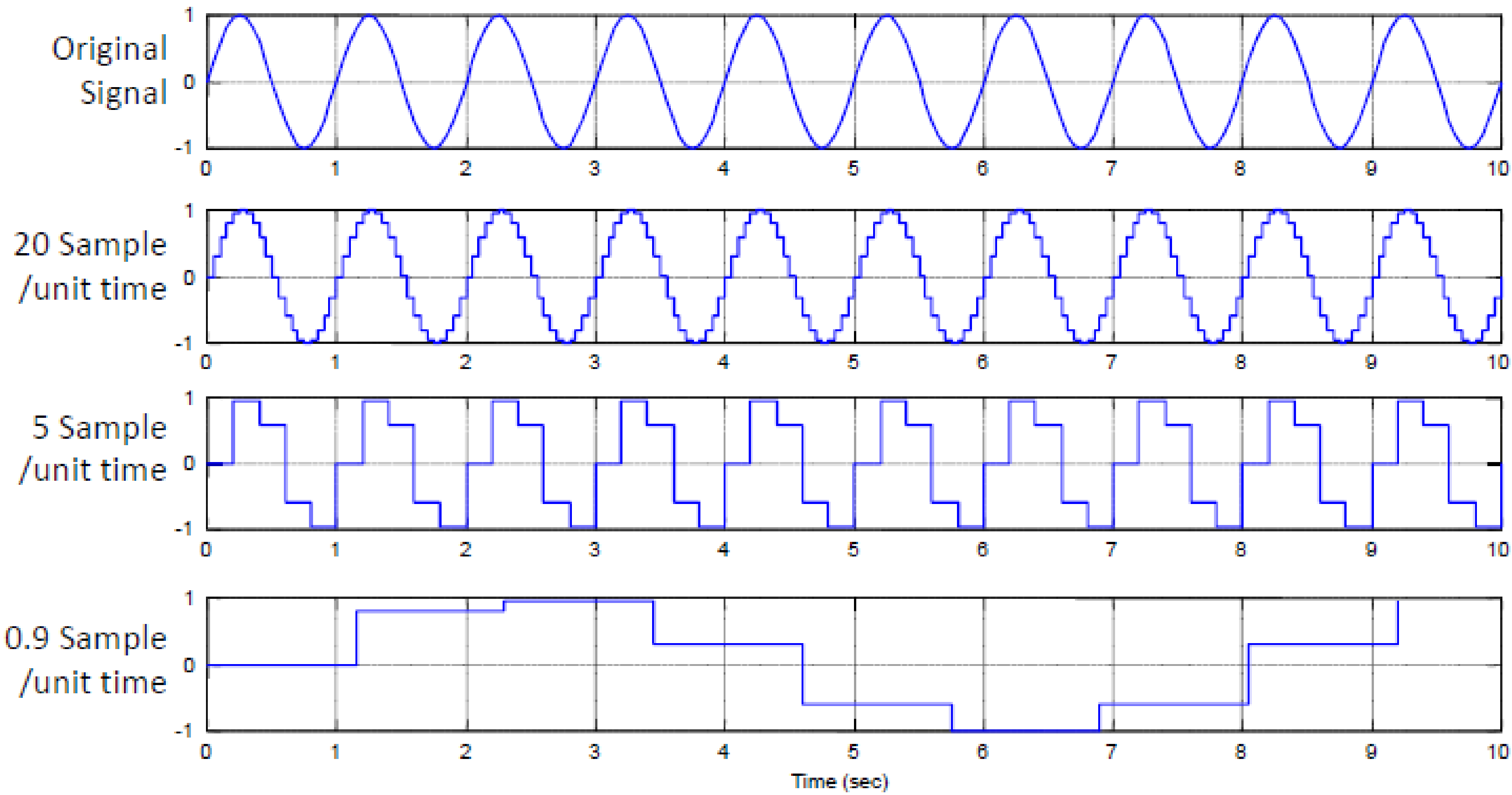
Approximation or quantizing error

Greater error = more noise

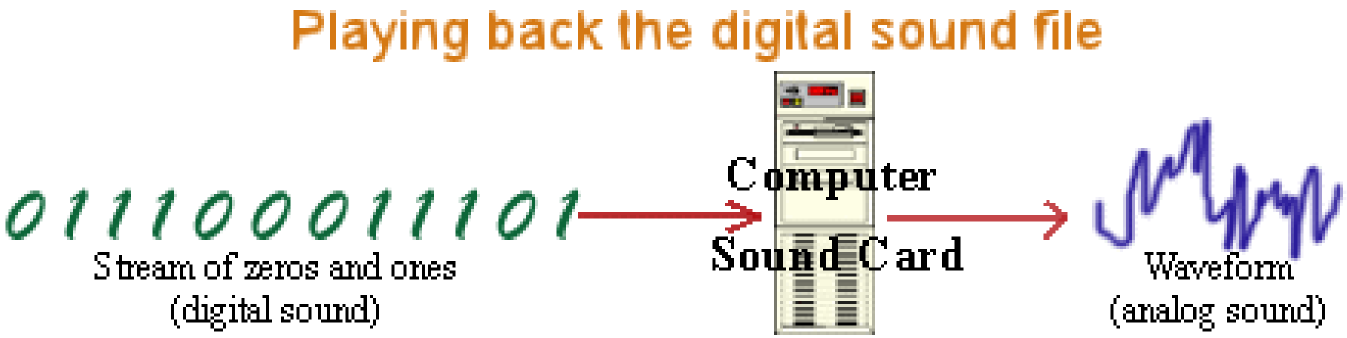
Example of Quantization Error

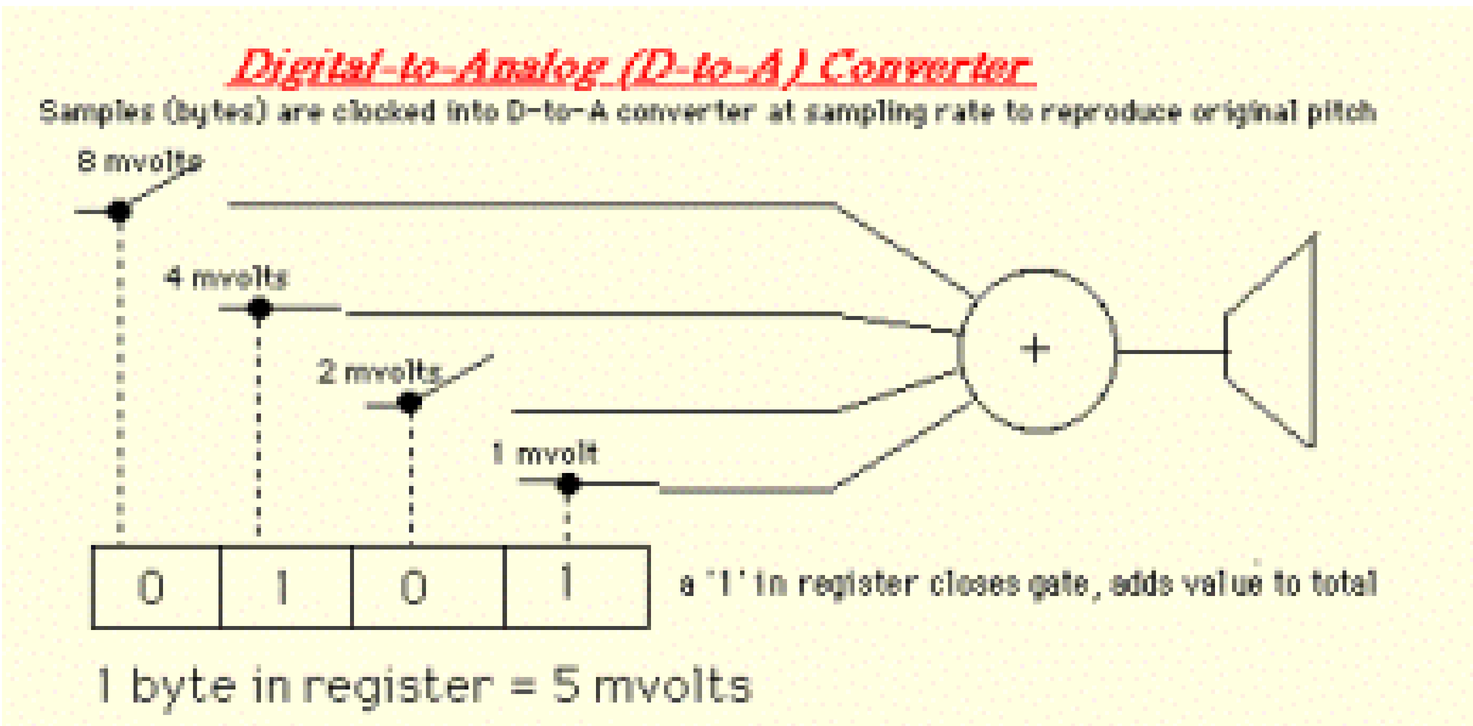


SAMPLING AND ALIASING



D/A Conversion





D/A Conversion

A_3	A_2	A_1	A_o	$8A_3+4A_2+2A_1+A_o$
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

CD ROMS

- Sampling rate is 44.1 kHz
- Nyquist Theorem says that the highest reproduced frequency is 22.05 kHz.
  - Any frequency above 22.05 kHz will produce aliasing
- A low pass filter is used to block frequencies above 22.05 kHz.

Problems with D/A

- Imperfect low pass filters
- Ideally you want 0 dB attenuation at 20 kHz going up to 90 dB at 22 kHz
  - Very expensive
- Oversampling will help
  - Sample at 8 X 20 kHz = 160 kHz
    - Then the low pass filtering needs to be accomplished in 140 kHz not 2 kHz (160 kHz sample rate – 20 kHz max range of hearing)

Problems with D/A

- Finite word length
  - Most systems today do 16 bit digitizing
  - 65536 different levels
- The loudest sounds need room, so the normal sounds don’t use the entire range
  - Problems occur at the low levels where sounds are represented by only one or two bits. High distortions result.
- Dithering adds low level broadband noise